Readers' Forum

Brief discussions of previous investigations in the aerospace sciences and technical comments on papers published in the AIAA Journal *are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one* Journal *page including formulas and figures. A Discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.*

Comment on "Formulas for Venting or Charging Gas from a Single Volume"

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Introduction

CLOSED-FORM solutions for the subcritical isothermal and isentropic venting problems at two values of the specific heat ratio, $k=7/5$ and $5/3$ were presented by Yang.[1] The purpose of this Comment is to show that more generalized solutions to these two problems exist[2] for a multiplicity of discrete k values ranging from 1-3.

Subcritical Isentropic Venting

Using the nomenclature of Ref. 1, the venting time is given by

$$t=(C_s/a)\int_{P_2/P_i}^{P_2/P_f}\left[\left(\frac{P_2}{P}\right)^{(k+3)/k}-\left(\frac{P_2}{P}\right)^{2(k+1)/k}\right]^{-1/2}\mathrm{d}\left(\frac{P_2}{P}\right),$$

$$\text{for } \frac{P_2}{P}\geq\left(\frac{2}{k+1}\right)^{k/(k-1)} \tag{1}$$

where C_s equals Eq. (9c) of Ref. 1, and

$$a=\left[\frac{2}{k-1}\left(\frac{k+1}{2}\right)^{(k+1)/(k-1)}\right]^{1/2}$$

[Note: In Ref. 1, the corresponding integral in Eq. (1) appears to have a typographical error wherein the exponent on the first pressure term is $(k-2)/2$ rather than the $(k+3)/k$ as shown herein. Also, the constant a listed in the Nomenclature has an error with $(k+1)/(k-1)$ appearing as a multiplier rather than as an exponent on the term $(k+1)/2$.]

The solution to Eq. (1) is achieved by introducing the substitution

$$Z=(P_2/P)^{(k-1)/k}$$

$$I_s=\left(\frac{k}{k-1}\right)\int_{Z_i}^{Z_f}Z^{-N}[1-Z]^{-1/2}\mathrm{d}Z \tag{2}$$

where $N=(k+1)/[2(k-1)]$.

Note that if N is an integer or integer plus one-half, then Eq. (2) can be readily integrated in closed form using the standard integration formula

$$I_s=\left(\frac{k}{k-1}\right)\left[\frac{-\sqrt{1-Z}}{(N-1)Z^{N-1}}+\frac{2N-3}{2(N-1)}\int\frac{\mathrm{d}Z}{Z^{N-1}\sqrt{1-Z}}\right] \tag{3}$$

where the last term in the recursion is given by

$$\int\frac{\mathrm{d}Z}{Z\sqrt{1-Z}}=\ell n(1-\sqrt{1-Z})/(1+\sqrt{1-Z})$$

or

$$\int\frac{\mathrm{d}Z}{Z^{3/2}\sqrt{1-Z}}=-2\frac{\sqrt{1-Z}}{\sqrt{Z}}$$

The conditional requirement on N can be stated as

$$N=(k+1)/[2(k-1)]=(i+2)/2$$

where

$$i=1,2,3,4,5,6,\text{ etc.}$$

This means that analytical solutions to Eq. 2 can be written for the following values of k:

$$k=(i+3)/(i+1)\quad\text{where } i=1,2,3,4,5,6,\text{ etc.}$$

or

$$k=2,\ 5/3,\ 6/4,\ 7/5,\ 8/6,\ 9/7,\ 10/8$$
$$11/9,\ 12/10,\ 13/11,\ 14/12,\text{ etc.}$$

For $k=5/3$ and $7/5$, Eq. (3) reduces to the identical expressions shown in Ref. 1 as Eqs. (6a) and (6b), respectively.

Subcritical Isothermal Venting

The venting time is given by

$$t=\left(\frac{C_t}{a}\right)\int_{P_2/P_i}^{P_2/P_f}\left(\frac{P_2}{P}\right)^{-1}\left[\left(\frac{P_2}{P}\right)^{2/k}-\left(\frac{P_2}{P}\right)^{(k+1)/k}\right]^{-1/2}\mathrm{d}\left(\frac{P_2}{P}\right) \tag{4}$$

where C_t equals Eq. (9d) from Ref. 1.

Again, introducing the substitution

$$Z=(P_2/P)^{(k-1)/k}$$

the integral portion of Eq. (4) reduces to

$$I_T=\left(\frac{k}{k-1}\right)\int_{Z_i}^{Z_f}Z^{-N}[1-Z]^{-1/2}\mathrm{d}Z \tag{5}$$

where $N=k/(k-1)$.

Equation (5) is in the same form as Eq. (2) and therefore can be solved using the integration formula of Eq. (3). The conditional on k for the isothermal problem is

$$k=(i+2)/i\quad\text{where } i=1,2,3,4,5,6,\text{ etc.}$$

or

$$k=3/1,\ 4/2,\ 5/3,\ 6/4,\ 7/5,\ 8/6$$
$$9/7,\ 10/8,\ 11/9,\ 12/10,\text{ etc.}$$

For the special case $k=5/3$, Eq. (5) reduces to the identical expressions shown in Ref. 1 as Eqs. (7a) and (17).

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For the case of $k=7/5$, the solution to Eq. (5) is

$$I_T=\frac{7}{15}\left\{\left[3\left(\frac{P_2}{P_i}\right)^{-4/7}+4\left(\frac{P_2}{P_i}\right)^{-2/7}+8\right]\left[\left(\frac{P_2}{P_i}\right)^{-2/7}-1\right]^{1/2}\right.$$
$$\left.-\left[3\left(\frac{P_2}{P_f}\right)^{-4/7}+4\left(\frac{P_2}{P_f}\right)^{-2/7}+8\right]\left[\left(\frac{P_2}{P_f}\right)^{-2/7}-1\right]^{1/2}\right\} \quad (6)$$

The corresponding solution for the nondimensional time parameter τ is written

$$\tau=\left(\frac{7}{2}\right)\ell n\left(\frac{5}{6}\right)+\frac{35\sqrt{5}}{648}\left\{\frac{428\sqrt{5}}{125}\right.$$
$$\left.-\left[3\left(\frac{P_2}{P}\right)^{-4/7}+4\left(\frac{P_2}{P}\right)^{-2/7}+8\right]\left[\left(\frac{P_2}{P}\right)^{-2/7}-1\right]^{1/2}\right\} \quad (7)$$

Equation (6) above does not agree with Eq. (7b) of Ref. 1. As a consequence, Eq. (7) above also differs from Eq. (19) of Ref. 1. A check of Eq. (6) was made by differentiating it and comparing the result of Eq. (5) evaluated at $k=7/5$. This comparison verified that Eq. (6) presented herein is the correct solution.

Acknowledgment

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References

[1]Yang, H. T., "Formulas for Venting or Charging Gas from a Single Volume," *AIAA Journal*, Vol. 24, Oct. 1986, pp. 1709–1711.

[2]Kato, H. T., "Transient Solutions for Venting and Charging a Constant Volume Chamber," Ford Aerospace and Communications Corp., TN ITN-2.17, April 26, 1968.

Reply by Author to H.T. Kato

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It is gratifying to see that more general solutions to the venting problem exist and are now widely available to the engineering and scientific community. We verified Eq. (6) of the Comment[1] by applying its recurrence formula (3).

Acknowledgment

I would like to thank Dr. Hamid Jafroudi for double checking the Comment in detail.

References

[1]Kato, H.T., Comment on "Formulas for Venting or Charging Gas from a Single Volume," *AIAA Journal*, Vol. 25, Sept. 1987, pp. 1273–1274.

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